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Spinors in Quantum Space with Torsion

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Dirac spinors are considered in quantized fiber-bundled spaces. It is shown that the spin connection has the same internal structure as in the Riemann–Cartan space as well as the quantized one. It is also assumed that the neutrino oscillation mechanism can be linked to the quantized and fibered character of the space at small distances.

1. INTRODUCTION

The study of the space-time structure at small distances is one of the main problems in the modern theory of elementary particles. Due to the success of the theory of supersymmetry (see, for example, De Witt, 1965; Salam and Strathdee, 1982; Scherk and Schwarz 1975) there arose a more or less confident supposition that space-time can not only be four-dimensional, but also many-dimensional; i.e., at the beginning of the development of the universe, space-time could have had its "true" 10 (or 11) dimensions, and thus the exact interaction symmetry could have been present. On the other hand, it is well known that the extension of space-time dimensions gives the possibility of unifying all the interactions including gravity. Such a theory has its problems, but their consideration is beyond the scope of this paper, and is studied in detail elsewhere (Aref'yeva and Volovich, 1985). The transition from higher temperature to the temperature of hadronization of quarks or symmetry is worked out, more or less in detail, for the case where the space has a fiber-bundle structure (see, for example, Bais and Batenburg, 1984). In the present paper, we attempt to explain the mechanism of spontaneous breaking of the symmetry, taking into account both the quantized and fibered character of the space. We also assume that the quantized character of space at small distances is an inalienable behavior of the given material object.

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The quantized character of space is considered by Dineykhan and Namsrai (1985; 1986a, b) as a small perturbation of the classical coordinates. The addition characterizing the quantum and fiber-bundle properties of space is a small stochastic perturbation and its magnitude is of a higher order than in the usual (classical) space. Proceeding from this, Dineykhan and Namsrai (1985, 1986a, b) introduced space quantum properties in the following way, as a small stochastic deviation from the classical coordinates:

$$x^{\mu} \Rightarrow \hat{x}^{\mu} = x^{\mu} + l\Gamma^{\mu}(x) \tag{1}$$

where the x^{μ} are the coordinates of usual (classical) space-time; l is a dimensional parameter characterizing the region of quantum properties of the space; $\Gamma^{\mu}(x)$ is determined from the tetrad field $e_a^{\mu}(x)$:

$$\Gamma^{\mu}(x) = \Gamma^{a} e_a^{\mu}(x)$$

and expresses the "internal" space that corresponds to every point of the usual (classical or world) space; and Γ^{a} is the generator of the symmetry group that acts in the internal space. For further concrete calculations we will use the generator of group $SU(2)$, i.e., the Pauli matrix instead of Γ^{a}. In Section 2 we consider the geometry of the space with coordinates $\hat{x}^{\mu}$, and Section 3 studies the Dirac spinors in the quantized space, obtains the equation for the tetrad field $e_a^{\mu}(\hat{x})$ in the space with coordinates $\hat{x}^{\mu}$, and also determines the spin connections. Finally it is shown that the mechanism of neutrino oscillation can be linked to the physics at very small distances, namely to the quantized as well as the fibered character of the space.

2. THE GEOMETRY OF THE SPACE WITH COORDINATES $\hat{x}^{\mu}$

We will show that the space with the coordinates $\hat{x}^{\mu}$ determined in (1) is fibered and the curved space with coordinates x^{μ} is its basis. For this purpose we consider the changes of the $\hat{x}^{\mu}$ coordinate of the fibered space under the usual Lorentz rotations of the base, i.e.,

$$\left.\begin{aligned} x^{\mu} \Rightarrow x'^{\mu} &= x^{\mu} + \delta x^{\mu} \\ \delta x^{\mu} &= \omega^{\mu}_{\nu} x^{\nu} \\ \omega^{\mu}_{\nu} &= \omega^{\mu\alpha} g_{\alpha\nu} \\ \omega^{\mu\nu} + \omega^{\nu\mu} &= 0 \end{aligned}\right\} \tag{2}$$

Then, taking into account the transformation of the tetrad field $e_a^{\mu}(x)$,

$$\delta e_a^{k}(x) = \omega^{k}_{n} e_a^{n}(x) - x^{m} \omega^{n}_{m} \partial_n e_a^{k}(x)$$

under the Lorentz rotations (2) and after simple calculations, we have

$$\delta \hat{x}^{k} = \omega^{k}_{n} \hat{x}^{n} - l x^{m} \omega^{n}_{m} \partial_n \Gamma^{k}(x)$$

Thus, the standard form (see, for example, De Witt, 1984) for the transformation of the coordinates of the fibered space corresponding to (2) is obtained. Making the necessary calculation and taking into account (1) and (2), one gets:

$$\delta\, d\hat{x}^k - d\delta\hat{x}^k = l\, \partial_n \Gamma^k(x)\, \omega^n_m\, dx^m \tag{3}$$

where d is the usual symbol of total differentiation. From the relation (3) one can see that the operations δ and d are noncommutative, though in the usual Riemann space they are commutative. Therefore the space with coordinates $\hat{x}^\mu$ determined in (1) is fibered and the curved space with the coordinates x^μ is its base.

The expression for the quantized coordinates $\hat{x}^\mu$ determined in (1) can be represented in the following form:

$$\hat{x}^\mu = \hat{U}(l)x^\mu$$

where $\hat{U}(l) = 1 + l\Gamma^\nu(x)\partial_\nu$ is the operator in the curved space. Using the determination of the momentum operator, i.e., $P_\nu = -i\partial_\nu$ (in the units such that $c = \hbar = 1$) for $\hat{U}(l)$, one can write

$$\hat{U}(l) = 1 + il\Gamma^\nu(x)P_\nu \approx \exp[il\Gamma^\nu(x)P_\nu] \tag{4}$$

Consequently, $\hat{U}(l)$ may be considered as a translation operator in the Riemann space with the translation vectors $\Gamma^\nu(x)$. In the general Lorentz transformations the translation vectors are commutative, but in our case the vectors $\Gamma^\nu(x)$ are noncommutative, thus creating the Clifford algebra: $\{\Gamma^\nu(x), \Gamma^\mu(x)\} = 2g^{\nu\mu}(x)$. It is obvious that the operators $\Gamma^\nu(x)P_\nu$ are commutative, satisfying the identity

$$\hat{U}^{-1}(l)\hat{U}(l) = \hat{U}(l)\hat{U}^{-1}(l) \equiv 1 \tag{5}$$

Hence, the operator

$$\hat{U}^{-1}(l) = \exp[-il\Gamma^\nu(x)P_\nu] \tag{6}$$

is an inverse operator to the operator $\hat{U}(l)$. From this it follows that we will change from usual space to the fibered one by the operator $\hat{U}(l)$ and the converse by $\hat{U}^{-1}(l)$. From the relations (4) and (5) it follows that the operator $\hat{U}(l)$ is a translation operator; therefore, its action on the function of the matter field can be determined in the standard form (see, for example, Gasiorowicz, 1966):

$$\hat{U}(l)\varphi(x)\hat{U}^{-1}(l) = \varphi(\hat{x})$$

where $\varphi(x)$ and $\varphi(\hat{x})$ are functions of the matter field in the usual and fibered spaces, respectively.

For the purpose of illustration let us consider the action of the operator $\hat{U}(l)$ on the following quantities:

(i) The partial derivatives, i.e., ∂_μ:

$$\hat{U}(l)\partial_\mu \hat{U}^{-1}(l) = [1 + il\Gamma^\alpha(x)P_\alpha]\partial_\mu[1 - il\Gamma^\beta(x)P_\beta]$$
$$= iP_\mu - il[\Gamma^\alpha(x), P_\mu]P_\alpha$$

and taking into account the action of the momentum operator P_μ i.e., $[f(x), P_\mu] = i\partial\mu f(x)$, we have

$$\hat{U}(l)\partial_\mu \hat{U}^{-1}(l) = \left(\delta^\alpha_\mu - l\frac{\partial\Gamma^\alpha(x)}{\partial x^\mu}\right) = \frac{\partial x^\alpha}{\partial \hat{x}^\mu}\frac{\partial}{\partial x^\alpha} \equiv \frac{\partial}{\partial \hat{x}^\mu} \tag{7}$$

On the other hand, the translation law of the gauge field $A_\nu(x)$ for the symmetry group $U(1)$ and the partial derivatives ∂_μ are equivalent. Then from (7) it follows that

$$\hat{U}(l)A_\mu(x)\hat{U}^{-1}(l) = \frac{\partial x^\nu}{\partial \hat{x}^\mu}A_\nu(x) \equiv A_\mu(\hat{x}) \tag{8}$$

(ii) Let us determine the $\gamma^\mu(\hat{x})$ Dirac matrix in the space with coordinates $\hat{x}^\mu$. Anologously to the partial derivatives, we have

$$\gamma^\mu(\hat{x}) \equiv \hat{U}(l)\gamma^\mu(x)\hat{U}^{-1}(l) = [1 + il\Gamma^\alpha(x)P_\alpha]\gamma^\mu(x)[1 - il\Gamma^\alpha(x)P_\alpha]$$
$$= \gamma^\mu(x) + il\Gamma^\alpha(x)[P_\alpha, \gamma^\mu(x)]$$

where $\gamma^\mu(x)$ is the Dirac matrix in the curved space and is written in the form

$$\gamma^\mu(x) = \gamma^a e^\mu_a(x)$$

where γ^a is the Dirac matrix in flat space, i.e., $\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab}$ and diag $\eta^{ab} = (+ - - -)$. Taking into account the action of the momentum operators, we obtain

$$\gamma^\mu(\hat{x}) = \gamma^\mu(x) + l\gamma^a\Gamma^b e^\alpha_b(x)\,\partial_\alpha e^\mu_a(x)$$

Let us now consider the expression $e^\alpha_b(x)\,\partial_\alpha e^\mu_a(x)$. Using the tetrad field $e^\mu_a(x) = \partial x^\alpha/\partial\xi^b$ (ξ^b are coordinates of flat space), and after some calculations, it can be shown that the following identity holds:

$$e^\alpha_b(x)\,\partial_\alpha e^\mu_a(x) = e^\alpha_a(x)\,\partial_\alpha e^\mu_b(x)$$

Consequently, for the Dirac matrix $\gamma^\mu(\hat{x})$ in the fibered space, we obtain

$$\gamma^\mu(\hat{x}) = \gamma^\nu(x)\,\partial_\nu\hat{x}^\mu \tag{9}$$

On the other hand, the Dirac matrix $\gamma^\mu(\hat{x})$ in the fibered space is presented in the standard way through the tetrad field $e^\mu_a(\hat{x})$:

$$\gamma^\mu(\hat{x}) = \gamma^a e^\mu_a(\hat{x})$$

where $e_a^\mu(\hat{x})$ is the tetrad field in the fibered space and can be written in the following way:

$$e_a^\mu(\hat{x}) \equiv \hat{U}(l) e_a^\mu(x) \hat{U}^{-1}(l) \tag{10}$$

Making some simple calculations, we have finally

$$e_a^\mu(\hat{x}) = e_a^\nu(x)\, \partial_\nu \hat{x}^\mu \tag{11}$$

From the formulas (7) and (9) it follows that the value $\gamma^\mu(x)\partial_\mu$ under the transformation from basis into fiber is invariant, i.e.,

$$\gamma^\mu(x)\, \partial/\partial x^\mu = \gamma^\mu(\hat{x})\, \partial/\partial \hat{x}^\mu$$

(iii) Now let us determine the metrical tensor $g^{\nu\mu}(\hat{x})$ in the space with the coordinates $\hat{x}^\mu$, given in (1). The contravariant metric tensor $g^{\nu\mu}(\hat{x})$, analogous to the contravariant vector, can be written in the following form:

$$g^{\nu\mu}(\hat{x}) \equiv \hat{U}(l) g^{\nu\mu}(x) \hat{U}^{-1}(l)$$

Using the definition of the metric tensor $g^{\nu\mu}(x)$ in the curved space though the tetrad field $e_a^\mu(x)$, i.e.,

$$g^{\nu\mu}(x) = e_a^\nu(x) e_b^\mu(x) \eta^{ab}$$

and taking into account the formulas (10) and (11), we have

$$g^{\nu\mu}(\hat{x}) = g^{\alpha\beta}(x)\, \partial_\alpha \hat{x}^\nu\, \partial_\beta \hat{x}^\mu \tag{12}$$

Dineykhan and Namsrai (1985) showed that the metrical tensor $g^{\nu\mu}(\hat{x})$ determined in (12) consists of two parts, symmetrical and antisymmetrical ones. In the space with the metrical tensor determined in (12), the torsion tensor is different from zero.

3. THE SPIN CONNECTION IN THE FIBERED SPACE

The Riemann space is a basis of the fibered space with coordinates $\hat{x}^\mu$ determined in (1). Therefore, let us consider some standard relations in the Riemann space, which will play an important role in our further calculations. The Dirac equation is

$$[i\gamma^\mu(x)\nabla_\mu - M]\psi(x) = 0$$

where M is the mass and $\psi(x)$ is the fermion state function; $\gamma^\mu(x)$ is the Dirac matrix in the curved space; ∇_μ is the convariant derivative, which is equal to

$$\nabla_\mu = \partial_\mu - \frac{i}{4}\omega_\mu^{ab}\sigma_{ab} + \left\{ \begin{matrix} \lambda \\ \nu\mu \end{matrix} \right\} I_\lambda^\nu(x) \tag{13}$$

where $I^{\nu}_{\mu}(x)$ is the generator of the translation group; $\{^{\lambda}_{\nu\mu}\}$ is the usual Christoffel symbol; σ^{ab} is the spin matrix; ω^{ab}_{μ} is the spin connection, which satisfies the following equation:

$$\partial_{\mu} e^{a}_{\nu}(x) - \omega^{a}_{\mu,b} e^{b}_{\nu}(x) + \begin{Bmatrix} \lambda \\ \nu\mu \end{Bmatrix} e^{a}_{\lambda}(x) = 0 \tag{14}$$

and the solution can be written as

$$\omega^{ab}_{\mu} = \tfrac{1}{2} e^{c}_{\mu}(x)(\Omega^{c}_{ab} - \Omega^{a}_{bc} - \Omega^{b}_{ac}) \tag{15}$$

where

$$\Omega^{c}_{ab} = e^{\mu}_{a}(x) e^{\nu}_{b}(x)[\partial_{\mu} e^{c}_{\nu}(x) - \partial_{\nu} e^{c}_{\mu}(x)]$$

If the transformation from basis (Riemann) space to fibered space is realized by the translation operator determined in (4), then the spinor field $\psi(\hat{x})$ in the fibered space can be expressed through the field $\psi(x)$ trivially (see, for example, Gasiorowicz, 1966):

$$\psi(\hat{x}) = \hat{U}(l)\psi(x)\hat{U}^{-1}(l)$$

From the Dirac equation in Riemann space, after some simple calculations, taking into account the relations (5), (7), and (9) we obtain

$$[i\gamma^{\mu}(\hat{x})\nabla_{\mu}(\hat{x}) - M]\psi(\hat{x}) = 0$$

where the $\gamma^{\mu}(\hat{x})$ are determined in (9) and

$$\nabla_{\mu}(\hat{x}) = \frac{\partial x^{\nu}}{\partial \hat{x}^{\mu}}\left(\partial_{\nu} - \frac{i}{4}\tilde{\omega}^{ab}_{\mu}\sigma_{ab}\right) + \Gamma^{\lambda}_{\mu\nu}(\hat{x}) I^{\nu}_{\mu}(\hat{x}) \tag{16}$$

where $\Gamma^{\lambda}_{\mu\nu}(\hat{x})$ is the affine connection and $I^{\nu}_{\lambda}(\hat{x})$ is the generator of the translation group in fibered space, respectively. Dineykhan and Namsrai (1985) showed, that the affine connection in the space with coordinates $\hat{x}^{\mu}$ determined in (1) can be written in the form

$$\Gamma^{\lambda}_{\nu\mu}(\hat{x}) = \begin{Bmatrix} \lambda \\ \nu\mu \end{Bmatrix} + g^{\lambda\gamma}(x)\frac{\partial x^{\alpha}}{\partial \hat{x}^{\rho}}\frac{\partial^2 \hat{x}^{\rho}}{\partial x^{\gamma}\,\partial x^{\mu}} g_{\alpha\nu}(x) \tag{17}$$

where $g^{\lambda\gamma}(x)$ is the metrical tensor in the Riemann space. $\tilde{\omega}^{ab}_{\mu}$ is the spin connection in the fibered space and similar to (14) satisfies the following equation:

$$\frac{\partial}{\partial \hat{x}^{\mu}} e^{a}_{\nu}(\hat{x}) - \frac{\partial x^{\rho}}{\partial \hat{x}^{\mu}}\tilde{\omega}^{a}_{\rho,b} e^{b}_{\nu}(\hat{x}) + \Gamma^{\lambda}_{\mu\nu}(\hat{x}) e^{a}_{\lambda}(x) = 0$$

After simple calculations using (10) and (17) we have

$$\tilde{\omega}^{a}_{\mu,b} = \begin{Bmatrix} \lambda \\ \nu\mu \end{Bmatrix} e^{a}_{\lambda}(x) e^{\nu}_{b}(x) - e^{\nu}_{b}(x)\partial_{\nu} e^{a}_{\mu}(x) + e^{\nu}_{b}(x) Q^{a}_{\nu\mu} \tag{18}$$

where $Q^a_{\nu\mu}$ is the torsion tensor obtained in Dineykhan and Namsrai (1985) and can be expressed through the affine connection $\Gamma^\lambda_{\nu\mu}(\hat{x})$ determined in (17) in the following way:

$$C^\lambda_{\mu\nu} \equiv e^\lambda_a(x) Q^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu}(\hat{x}) - \Gamma^\lambda_{\nu\mu}(\hat{x})$$

Introducing the new operator determined in the form

$$\Delta = \gamma^\mu(\hat{x})\, \nabla_\mu(\hat{x}) - \gamma^\mu(\hat{x}) \Gamma^\lambda_{\mu\nu}(\hat{x}) I^\nu_\lambda(\hat{x})$$

and taking into account the relations (9), (16), and (18), we obtain after some simple calculations

$$\Delta = \gamma^\mu(x)(\partial_\mu - \tfrac{1}{4} i\omega^{ab}\sigma_{ab}) - \tfrac{1}{4} i\gamma^\mu(x) e^a_\alpha(x) e^b_\beta(x)\, Y^{\alpha\beta}_\mu \sigma_{ab} \tag{19}$$

where $Y^{\alpha\beta}_\mu$ is the contorsion tensor and is equal to (see, for example, Yajima and Kimura, 1985)

$$Y^\lambda_{\mu\nu} = \tfrac{1}{2}(C^\lambda_{\mu\nu} + C^\lambda_{\mu\nu} + C^\lambda_{\nu\mu}) \tag{20}$$

We use the following representation for the Dirac matrix γ_5 in the curved space:

$$\gamma_5 = -\frac{i}{4!} E^{\alpha\beta\nu\mu} \gamma_\alpha(x) \gamma_\beta(x) \gamma_\nu(x) \gamma_\mu(x) \tag{21}$$

where

$$E_{\alpha\beta\nu\mu} = e\mathscr{E}_{\alpha\beta\nu\mu}, \qquad \mathscr{E}_{0123} = 1$$

and $e = \det(e^\mu_a)$.

Calculations taking into account (21) give from (19) the following expression for Δ:

$$\Delta = \gamma^\mu(x)[\partial_\mu - \tfrac{1}{4} i\omega^{ab}\sigma_{ab} + \tfrac{1}{4}\gamma_5 A_\mu(x)] \tag{22}$$

where

$$A_\mu(x) = E_{\mu\alpha\beta\nu} Y^{\alpha\beta\nu} \tag{23}$$

and $A_\mu(x)$ is a chiral gauge field for the $U(1)$ group in the curved space. Studying the Riemann-Cartan space, Yajima and Kimura (1985) obtain an analogous expression for the operator Δ determined in (22). Therefore it can be assumed that the spin connection has similar internal structure in the Riemann-Cartan space and as well as in the space with the coordinates $\hat{x}^\mu$ determined in (1).

For the sake of completing the picture, we consider the Rarita-Schwinger spinor. It is common knowledge that under the general Lorentz transformation the Rarita-Schwinger spinor changes simultaneously as a vector and spinor. Therefore, taking into account (8), we can write the spinor field with spins $\frac{3}{2}$ in the fibered space in the form

$$\psi_\mu(\hat{x}) \equiv \hat{U}(l)\psi_\mu(x)U(l) = \frac{\partial x^\nu}{\partial \hat{x}^\mu}[\psi_\nu(x) + l\Gamma^\alpha(x)\partial_\alpha \psi_\nu(x)]$$

where $\psi_\nu(x)$ and $\psi_\nu(\hat{x})$ are Rarita-Schwinger spinors in the curved -basis) and fibered spaces, respectively.

4. ON THE OSCILLATION OF THE NEUTRINO

The Dirac equation for massless particles is written in the standard form (see, for example, Hehl *et al.*, 1976; Obukhov 1983) by taking into account the torsion tensor:

$$\gamma^\mu(\hat{x})[\nabla_\mu(\hat{x}) - \Gamma^\lambda_{\mu\nu}(\hat{x})I^\nu_\lambda(\hat{x})]\psi(\hat{x}) = 0$$

where $\nabla_\mu(\hat{x})$ and $\Gamma^\lambda_{\mu\nu}(\hat{x})$ are determined in (16) and (17), respectively; $\psi(\hat{x})$ is the neutrino field in the fibered space. Using (19) and (22), we have

$$[\gamma^\mu(x)\nabla_\mu - \gamma^\mu(x)\gamma_5 A_\mu(x)]\psi(x) = 0 \tag{24}$$

where $\gamma^\mu(x)$ is the Dirac matrix and $\psi(x)$ is the neutrino field in the curved space; $\nabla_\mu = \partial_\mu - \frac{1}{4}i\omega^{ab}_\mu \sigma_{ab}$ and $A_\mu(x)$ is the chiral gauge field determined in (23). Then the formula (24) represents the Dirac equation for the neutrino in the usual curved space with torsion. In the framework of the standard spinor theory the second term in equation (24) must be the mass of a given particle with the state function $\psi(x)$. Consequently, the neutrino in the quantized and fibered space has a mass the value of which is equal to the second term in equation (24). On the other hand, it has been shown (Dineykhan, 1986a, b) that, if the space is quantized and fibered, then the motion of a particle in the vacuum is equivalent to motion in the medium, leading to the effect of charge screening and also the creation of a frinction force conditioned by its nature against extermal forces, including the gravity force. These effects are consequences of the quantized character of the space at small distances. If the motion of a particle in the quantized space is equivalent to the motion in the medium, then the mass of the neutrino in the quantized space is not equal to zero and consequently the neutrino is considered as a massive particle in the medium. If the mass of the neutrino is not zero in the medium, then neutrino oscillation occurs.

We emphasize that the neutrino oscillation mechanism can thus be linked to the quantized and fibered character of the space at small distances.

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